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Dear Colleagues,

Re: Dr Jorge L. P. Ramírez Alfonsín's work submitted for *Habilitation à diriger des recherches*

I strongly and without reservation support the award of *Habilitation à diriger des recherches* to Dr Ramírez Alfonsín on the basis of the work submitted to the panel and summarised in the dissertation *Geométrie Combinatoire, Théories de Nombres et Graphes*.

Dr Ramírez Alfonsín's dissertation contains some first class results which reveal his remarkable problem-solving skills. It also shows his wide and diverse research interests. But what I find even more important, Dr Ramírez Alfonsín is achieving in his work a natural and fruitful synthesis of techniques and ideas from different branches of mathematics.

At this point, I wish to take the liberty of making a few general remarks concerning the underlying principles of assessment of academic achievement and academic progress. I believe that we have to put special value on work of those of our colleagues who, like Dr Ramírez Alfonsín, strive to work in several adjacent areas, or at a junction of various branches of mathematics. I can offer two reasons for my preference. First, it is difficult to be a polymath. Secondly, interdisciplinary research supports the unity and the integrity of mathematics. This second aspect of interdisciplinarity is increasingly important because specialisation, fragmentation and compartmentalisation are the prevalent tendencies in the intellectual and technological development of the modern world.

I find it very important that Dr Ramírez Alfonsín pays special attention to problems which play an integrating role in mathematics. His work on the Frobenius Problem is a wonderful example of this approach. I quote Dr Ramírez Alfonsín's book [1].

During the early part of the last century, Ferdinand Georg Frobenius (1849–1917) raised the following problem, known as the Frobenius Problem: given relatively prime positive integers a_1, \dots, a_n , find the largest natural number (called the *Frobenius number* and denoted by $g(a_1, \dots, a_n)$) that is not

¹J.L. Ramírez Alfonsín, **The Diophantine Frobenius Problem**. Oxford University Press, 2005, 205 pp. ISBN 0198568207.

representable as a nonnegative integer combination of a_1, \dots, a_n . At first glance the Frobenius Problem may look deceptively specialized. Nevertheless it crops up again and again in the most unexpected places and has been extremely useful in investigating many different problems.

Théorème 1.3.1 of Dr Ramírez Alfonsín's Dissertation states the \mathcal{NP} -hardness of the Frobenius Problem under Turing reductions [2]. It is a very important result. Dr Ramírez Alfonsín proves Theorem 1.1 by giving a short and elegant Turing reduction from the Integer Knapsack Problem [3, p. 376].

The difficulty and the beauty of the Fibonacci Problem for specific semigroups of numbers is demonstrated by Théorème 1.3.2 of the Dissertation [4]. Then the Fibonacci Problem is applied to study of certain types of tilings of squares and tori by bricks [5, Théorème 1.3.5]; the condition for the existence of a tiling is given in terms of the Frobenius number for a certain instance of the Frobenius Problem.

The Dissertation also contains a number of deep and beautiful results from discrete and combinatorial geometry and graph theory. For example, any embedding into \mathbf{R}^3 of the complete graph K_7 on 7 vertices contains the trefoil knot or its mirror image [6; Théorème 2.3.1]. I find it very attractive that the proof of this result uses the theory of oriented matroids.

The ideology of Dr Ramírez Alfonsín's approach to spatial embeddings of graphs is reflected in his illuminating survey [7]. It is naturally linked to the study of the partitions of the sets of edges of the complete graphs into paths [8; Théorèmes 3.1.1–3.1.3] and trees [9; Théorème 3.2.1], to the theory of oriented matroids [10; Théorèmes 2.1.1 and 2.1.2] and [11; Théorème 2.1.5] and hyperplane arrangements [12; Théorème 2.2.1], [13; Théorème 2.2.4], [14; Théorème 2.2.4], [15; Théorème 2.2.6].

²J.L. Ramírez Alfonsín, Complexity of the Frobenius problem, *Combinatorica* 16 (1996) 143–147.

³C. H. Papadimitrou and K. Steiglitz, **Combinatorial Optimization: Algorithms and Complexity**, Prentice-Hall, 1982.

⁴J. M. Marin, J. L. Ramírez Alfonsín and M. P. Revuelta, On the Frobenius number of Fibonacci numerical semigroups, *INTEGERS* 7 (2007) # A14.

⁵D. Labrousse and J.L. Ramírez Alfonsín, Tiling rectangles and torus. *Discrete and Computational Geometry*, soumis: <http://www.ecp6.math.jussieu.fr/pageperso/ramirez/habilitation/tiles.PDF>.

⁶J. L. Ramírez Alfonsín, Spatial graphs and oriented matroids: the trefoil, *Discrete and Computational Geometry* 22 (1999) 149–158.

⁷J. L. Ramírez Alfonsín, Knots and links in spatial graphs: A Survey, *Discrete Mathematics* 302 (1–3) (2005) 225–242.

⁸J.L. Ramírez Alfonsín, The spread of K_n , *Discrete Mathematics* 175 (1997) 221–229.

⁹J. L. Ramírez Alfonsín, Gracefulness of replicated paths and cycles, *Ars Combinatoria* 53 (1999) 257–269.

¹⁰D. Forge and J. L. Ramírez Alfonsín, Connected coverings and an application to oriented matroids, *Discrete Mathematics* 187 (1998) 109–121.

¹¹D. Forge, J. L. Ramírez Alfonsín and H. Yeun, Disconnected coverings for oriented matroids via simultaneous mutations, *Discrete Mathematics*, 258 (2002) 353–359.

¹²J. L. Ramírez Alfonsín, Lawrence oriented matroids and a problem of McMullen on projective equivalences of polytopes, *European Journal of Combinatorics* 22 (2001) 723–731.

¹³D. Forge and J. L. Ramírez Alfonsín, Straight line arrangements in the real projective plane, *Discrete and Computational Geometry* 20 (1998) 155–161.

¹⁴D. Forge and J. L. Ramírez Alfonsín, On counting the k -face cells of cyclic arrangements, *European Journal of Combinatorics* 22 (2001) 307–312

¹⁵J. L. Ramírez Alfonsín, Cyclic arrangements and Roudneff's conjecture in the space, *Information Processing Letters* 71 (1999) 179–182.

To summarise, the Dissertation represents very important mathematical work which meets all criteria for *Habilitation à diriger des recherches*.

I reiterate that I wholeheartedly support the award of *Habilitation à diriger des recherches* to Dr Ramírez Alfonsín.

Should you wish to ask me any further questions regarding Dr Ramírez Alfonsín's work please do not hesitate to contact me.

Yours sincerely,

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