Report on
Géométrie Combinatoire, Théorée des Nombres et Graphes
submitted by
Jorge Ramirez Alfonsin.

This is a very impressive piece of work consisting (as the title suggests) of three distinct chapters. I will deal with these in the order in which they are presented.

The first chapter on the elementary theory of numbers is a development of a long term interest of the author. In the days when he was doing his doctorate at Oxford he was working on the so-called 'jugs of wine' problem. I have to admit that this was a problem which I never regarded seriously, but I now admit that I was wrong and that Ramirez Alfonsin's work in this area, and in particular its development into the classical Frobenius problem is very impressive indeed. His book *The Diophantine Frobenius Problem* published (2005) is a masterly, authoritative piece of work. On the purely research side, his (1996) paper proving the NP-completeness of the Diophantine Frobenius problem is extremely important, as evidenced by its publication in the most prestigious combinatorics journal *Combinatorics*.

I turn now to Chapter 2 on Discrete and Convex Geometry. This is extremely impressive. The study of oriented matroids demands not only a strong background in matroid theory, but also a very good geometric intuition. Most matroid theorists are afraid of this area. Working alone and also with Forge he has obtained a series of good results and new bounds in what I regard as a very difficult area. What is particularly impressive is that not only has he obtained new results in the theory of oriented matroids per se, but he has also applied his knowledge of this area to make significant contributions in classical parts of convex geometry. A good example of this is a problem posed by Peter M^{c}Mullen where Ramirez Alfonsin improves results of David Larman. I know both M^{c}Mullen and Larman well, they are extremely clever professors at University College, London. To improve their results in an area in which they are recognised experts is a considerable achievement. I know that Larman was very impressed by Ramirez Alfonsin's result and arranged for him to be invited to UCL to discuss related problems.

In the same way, his papers with Forge in which they answer positively a question posed by Grunbaum about arrangements, and his work [7] and [9] answering open questions on cyclic arrangements of hyperplanes are noteworthy contributions in a very difficult problem solving area.

The third aspect of his geometric work is concerned with spatial representations of knots. Again this is a difficult area with few significant results since Conway and M^{c}Gordon in 1983 showed that K_6 could not be linklessly embedded in space and that K_7 was always knotted in space. He has written (2005) an extremely useful survey of this area and has produced significant new results, in particular showing that any spatial representation of K_7 must contain either the trefoil or its mirror image. Also, very recently he has attacked successfully some related problems concerning knots and cyclic polytopes. Overall, this second chapter contains an impressive array of research achievements.

The final chapter of the thesis is concerned with four seemingly unrelated problems about graphs. All four are hard problems. As far as I am aware the concept of 'spread' which underlies the first problem has not received wide attention. Although it does not seem to have any 'outside motivation' it is an attractive concept. A particularly attractive question posed by Ramirez Alfonsin is whether spread of the K_n/n converges to 1. It's not even clear to me that one can prove the existence of the limit and the original problem posed could be open for a long time. The second problem about 'graceful labellings' has quite a long history. Classifying which graphs have a graceful labelling is, I am fairly sure, a quite hopeless task. As a result the best that can be hoped for is a series of ad hoc results such as are achieved here. The third problem, posed by Alspach in 1981, offers a little more hope, but again, all that is known so far are partial results such as Ramirez Alfonsin presents here. The final problem studied concerns embeddings of the combinohedron and is, in my view, the most attractive part of this chapter. Although not all the problems of this chapter are to my taste I can confidently say that they are genuinely difficult problems and I can see their appeal to a problem solver.

Turning now to my overall assessment, I have no hesitation whatsoever in saying that this dissertation represents an extremely impressive body of work. It is well above the standard you set, namely "corresponding to a tenure position in an American university".

The candidate has shown a quite remarkable breadth in his written work. To be able to make very significant contributions in areas ranging from complexity through combinatorics to arrangements of hyperplanes and cyclic polytopes is truly impressive. It is also rare.

Apart from the results obtained the thesis contains a host of very appealing research problems and conjectures. This quality of being able to suggest good problems will be

invaluable as a supervisor of research students. I have also heard Ramirez Alfonsin give several research seminars. Each time he has been excellent.

To sum up, I unreservedly recommend the acceptance of this dissertation as being well above the standard required.

Dominic Welsh